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ON DERIVATIVE AND PROPORTIONAL FEEDBACK DESIGN FOR DESCRIPTOR SYSTEMS

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On Derivative and Proportional Feedback Design for Descriptor Systems

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Abstract

For linear time-invariant singular systems (continuous or discrete) it is customary to use a proportional state or output feedback control in order to achieve a desired closed loop behaviour. Derivative feedback is rarely considered. In this paper we examine how derivative feedback in descriptor systems can be used to alter the structure of the system pencil under various controllability conditions. It is shown that derivative and proportional feedback controls can be constructed (by numerically reliable methods) such that the closed loop system has a given form and is also regular. This property ensures the solvability of the resulting system of dynamic-algebraic equations.

Applications are also presented. For the linear-quadratic optimal control problem, conditions are given under which a derivative feedback transforms a singular problem into a standard L-Q problem, leaving the cost functional positive-definite. We discuss the problem of pole placement with derivative feedback alone and in combination with the usual proportional state feedback.

1 Introduction

We consider linear time-invariant (continuous or discrete) dynamical systems of the form

$$E dx/dt = Ax(t) + Bu(t), \qquad x(t_0) = x_0 \tag{1}$$
$$y(t) = Cx(t), \tag{2}$$

or

$$Ex_{k+1} = Ax_k + Bu_k, \qquad x_0 \text{ given} \tag{3}$$
$$y_k = Cx_k, \tag{4}$$

where $E, A \in \mathbf{R}^{n\times n}$, $B \in \mathbf{R}^{n\times m}$, $C \in \mathbf{R}^{p\times n}$ and $\operatorname{rank} B = m \leq n$, $\operatorname{rank} C = p \leq n$. Here $x(t)$ or $x_k \in \mathbf{R}^n$ is the state, $y(t)$ or $y_k \in \mathbf{R}^p$ is the output, and $u(t)$ or $u_k \in \mathbf{R}^m$ is the input or control of the system. Such systems are called *descriptor* or *generalized state-space* systems. In the case $E = I$, the identity matrix, we refer to (1)-(2) and (3)-(4) as *standard* systems.

Descriptor systems arise naturally in a variety of practical circumstances [31,22] and have recently been investigated in a number of papers [1,2,3,6,7,8,9,10] [11,12,13,14,18,19,20,21,23,24,25,26,27,28] [29,30,32,33,34,35,36,37,38,39]. The response of a descriptor system can be described in terms of the eigenstructure of the matrix pencil

$$\alpha E - \beta A. \tag{5}$$

In order to alter the behaviour of the system, it is customary to use proportional state or output feedback to modify the matrix A. The closed loop system pencil then becomes

$$\alpha E - \beta(A + BFC), \tag{6}$$

where the control is taken to be $u = Fy + v$ or $u_k = Fy_k + v_k$. In the theory of matrix pencils, the roles of E and A are interchangeable, but the analogous use of derivative state or output feedback in multivariable systems has received little attention in the literature. Derivative feedback modifies the matrix E, and the closed loop system pencil then becomes

$$\alpha(E + BGC) - \beta A, \tag{7}$$

where the control is taken to be $u = -G\dot{y} + v$ or $u_k = -Gy_{k+1} + v_k$.

It has long been recognized that derivative feedback is an essential tool in practical control system design, and recently it has been shown that discrete-time observers, using both current and past information to obtain a system pencil of the form

$$\alpha(E + BGC) - \beta(A + BFC), \tag{8}$$

can give improved state estimates [6]. In this paper we consider both derivative and proportional feedback and examine the properties that can be achieved with these types of feedback in various applications. In particular we discuss the pole placement problem and the linear quadratic regulator problem. In Section 2 we introduce notation and some preliminary results. In Section 3 we summarize the mathematical properties that can be achieved for pencils of the forms (6),(7),and (8) by suitable choices of F and G. The applications are discussed in Sections 4 and 5, and concluding remarks are given in Section 6. Details of the results presented here can be found in [4].

2 Definitions and Preliminaries

2.1 Eigenstructure of Descriptor Systems

The system equations (1) and (3) are said to be *solvable* if and only if the system pencil (5) is *regular*, that is

$$\det(\alpha E - \beta A) \not\equiv 0 \quad \forall(\alpha,\beta) \in \mathbf{C}^2 \backslash \{0,0\} \tag{9}$$

(See [5,38].) The behaviour of the system response $x(t)$ or x_k is then governed by the eigenstructure of the system pencil. For a regular pencil the *generalized eigenvalues* are defined to be the pairs $(\alpha_j,\beta_j) \in \mathbf{C}^2$ such that

$$\det(\alpha_j E - \beta_j A) = 0, \qquad j = 1,2,...n. \tag{10}$$

Eigenvalue pairs (α_j,β_j) where $\beta_j \neq 0$ are said to be *finite* and, without loss of generality, can be taken to have the 'value' $\lambda_j = \alpha_j/\beta_j$. Pairs where $\beta_j = 0$ are said to be *infinite* eigenvalues. The maximum number of finite eigenvalues which a pencil can have is less than or equal to the rank of E. (For a pencil which is *not* regular, the generalized eigenvalues can be similarly defined as the pairs (α_j,β_j) such that the pencil loses rank.)

For regular pencils the solution of the system equations can be characterized in terms of the Kronecker Canonical Form (KCF) [16]. In this case there exist non-singular matrices X and Y (representing the right and left generalized eigenvectors and principal vectors of the system pencil, respectively) which transform E and A into the KCF :

$$Y^T E X = \begin{bmatrix} I & 0 \\ 0 & N \end{bmatrix}, \qquad Y^T A X = \begin{bmatrix} J & 0 \\ 0 & I \end{bmatrix}. \tag{11}$$

Here J is a Jordan matrix corresponding to the finite eigenvalues of the pencil and N is a nilpotent matrix such that $N^m = 0, \;\; N^{m-1} \neq 0$, also in Jordan form corresponding to the infinite eigenvalues. The *index* of the system is defined to be equal to the degree m of nilpotency. (For pencils which are not regular, the KCF and the index of the system can be defined similarly. See [4].)

We observe that a descriptor system is regular and index 0 if and only if E is non-singular. In this case the system can be reformulated as a standard system and the usual theory applies. In practice the reduction to standard form can be numerically unstable, however, (if E is ill-conditioned!) and, hence, even for index 0 systems, it is preferable to work directly with the generalized state-space form.

We observe also that a descriptor system is regular and index ≤ 1 if and only if it has exactly $q = \operatorname{rank} E$ finite eigenvalues. Conditions for the system to be regular and index ≤ 1 are given in the following lemma [18].

Lemma 1 *Let E, $A \in \mathbf{C}^{n\times n}$. Let S_∞ and T_∞ be full rank matrices whose columns span the null spaces $\mathcal{N}(E)$ and $\mathcal{N}(E^H)$, respectively. Then the following are equivalent:*

(i) $\alpha E - \beta A$ *is regular and index* ≤ 1

(ii) $\operatorname{rank}[E, AS_\infty] = n$

(iii) $\operatorname{rank}\begin{bmatrix} E \\ T_\infty^H A \end{bmatrix} = n.$

For systems which are regular and index ≤ 1, there exists a unique solution for all admissible controls which satisfy certain initial consistency conditions. For higher index systems, impulses can arise in the response of the system if the control is not sufficiently smooth [36]. It is, therefore, desirable to select a feedback which ensures that the closed loop system is regular and index ≤ 1 if possible.

2.2 Controllability and Observablilty of Descriptor Systems

The definitions of controllablility and observability for standard control systems can be extended to descriptor systems. Various types of controllablility/observability can be identified, however [38]. Here we investigate the properties of the generalized state-space systems (1)-(2) and (3)-(4) under the following conditions.

Definition 2 *Let $\alpha E - \beta A$ be a regular pencil. Then the triple (E, A, B) and the corresponding descriptor system are said to be* completely controllable *(C-controllable) if and only if*

$$\mathbf{C0} : \operatorname{rank}[\alpha E - \beta A, B] = n, \quad \forall(\alpha,\beta) \in \mathbf{C}^2\backslash\{0,0\}. \tag{12}$$

Similarly the triple (E, A, C) and the corresponding descriptor system are said to be completely observable *(C-observable) if and only if*

$$\mathbf{O0} : \operatorname{rank}\begin{bmatrix} \alpha E - \beta A \\ C \end{bmatrix} = n, \quad \forall(\alpha,\beta) \in \mathbf{C}^2\backslash\{0,0\}. \tag{13}$$

We remark that a system is completely controllable and/or completely observable only if

$$\operatorname{rank}[E, B] = n \quad \text{and/or} \quad \operatorname{rank} \begin{bmatrix} E \\ C \end{bmatrix} = n. \tag{14}$$

Complete controllability ensures that for any given initial and final states $x_0, x_f \in \mathbb{R}^n$ of the system, there exists an admissible control which transfers the system from x_0 to x_f in finite time [38]. Hence descriptor systems which are completely controllable can be expected to have similar properties to standard systems. Analogous remarks hold for completely observable systems.

Weaker definitions of controllability and observability are given by the following.

Definition 3 *Let $\alpha E - \beta A$ be a regular pencil. Then the triple (E, A, B) and the corresponding descriptor system are said to be* strongly controllable *(S-controllable) if and only if*

$$\begin{array}{ll} \mathbf{C1}: \operatorname{rank}[\lambda E - A, B] = n, & \forall \lambda \in \mathbb{C}; \\ \mathbf{C2}: \operatorname{rank}[E, AS_\infty, B] = n, & \text{where the columns of } S_\infty \text{ span } \mathcal{N}(E). \end{array} \tag{15}$$

Similarly the triple (E,A,C) and the corresponding descriptor system are said to be strongly observable *(S-observable) if and only if*

$$\begin{array}{ll} \mathbf{O1}: \operatorname{rank} \begin{bmatrix} \lambda E - A \\ C \end{bmatrix} = n, & \forall \lambda \in \mathbb{C}; \\ \mathbf{O2}: \operatorname{rank} \begin{bmatrix} E \\ T_\infty^H A \\ C \end{bmatrix} = n, & \text{where the columns of } T_\infty \text{ span } \mathcal{N}(E^H). \end{array} \tag{16}$$

We remark that C-controllablity and C-observability imply S-controllability and S-observability, respectively. Clearly conditions **C1** and **O1** follow from **C0** and **O0**, respectively, for $\beta \neq 0$ and $\lambda = \alpha/\beta$. Conditions **C2** and **O2** follow from (14), but are weaker. In the literature, systems which satisfy **C2** (or **O2**) are often described as "controllable (or observable) at infinity" [10,18,36].

We remark also that the properties of controllability and observability of descriptor systems are preserved under certain transformations. Specifically, conditions **C0**, **C1**, **C2**, **O0**, **O1** and **O2** are all preserved under non-singular transformations of the pencil and under proportional state and output feedback. With the exception of conditions **C2** and **O2**, these same conditions are also preserved under derivative state and output feedback. We have the following [4].

Lemma 4 *Let (E, A, B) satisfy the condition* **C0** *or* **C1** *or* **C2**, *and let (E, A, C) satisfy the condition* **O0** *or* **O1** *or* **O2**. *Then for any P and $Q \in \mathbb{R}^{n \times n}$ which are non-singular and for any $F \in \mathbb{R}^{m \times p}$, the systems*

$$\tilde{E} = PEQ, \quad \tilde{A} = PAQ, \quad \tilde{B} = PB, \quad \tilde{C} = CQ \tag{17}$$

and

$$\tilde{E} = E, \quad \tilde{A} = A + BFC, \quad \tilde{B} = B, \quad \tilde{C} = C. \tag{18}$$

also satisfy these conditions.

Furthermore, for any matrix $G \in \mathbf{R}^{m \times p}$, *the system*

$$\tilde{E} = E + BGC, \quad \tilde{A} = A, \quad \tilde{B} = B, \quad \tilde{C} = C. \tag{19}$$

also satisfies these conditions with the exception of **C2** *and* **O2**.

3 Derivative and Proportional Feedback for Descriptor Systems

In this section we discuss conditions under which we can alter the structure of system pencil (5) by the use of derivative and/or proportional feedback. We show first that if the triples (E, A, B) and (E, A, C) are C-controllable and C-observable, respectively, then the system (1)- (2) or (3)-(4) can be transformed into a *standard* system by derivative feedback.

Theorem 5 *There exists a matrix* $G \in \mathbf{R}^{m \times p}$ *such that the system matrix* $E + BGF$ *is* non-singular *if and only if (14) holds.*

Proof: See [4]

Corollary 6 *There exists a feedback control* $u = -G\dot{y} + v$ *or* $u_k = -Gy_{k+1} + v_k$ *such that the system matrix* $E + BGC$ *is non-singular and the closed loop system defined by the triples* $(E+BGC, A, B)$ *and* $(E+BGC, A, C)$ *is C-controllable and C-observable if and only if the triples* (E, A, B) *and* (E, A, C) *are C-controllable and C-observable, respectively.*

We remark that if $S \equiv E + BGC$ is non-singular, then under derivative feedback the corresponding closed loop system is equivalent to the standard system

$$\dot{x} = \tilde{A}x + \tilde{B}v, \tag{20}$$

or

$$x_{k+1} = \tilde{A}x_k + \tilde{B}v_k \tag{21}$$

where $\tilde{A} = S^{-1}A$, $\tilde{B} = S^{-1}B$. Furthermore, the feedback can be implemented directly in terms of the states and the external inputs of the system. Using the closed loop form (20) we find that

$$u = -G\dot{y} + v = Wx + Vv \tag{22}$$

or

$$u_k = -Gy_{k+1} + v_k = Wx_{k+1} + Vv_k \tag{23}$$

where

$$W = -GCS^{-1}A \quad \text{and} \quad V = I - GCS^{-1}B. \tag{24}$$

We now show that if the system is S-controllable and S-observable, then a closed loop system pencil which is regular and index ≤ 1 can be obtained by proportional *or* derivative feedback. We have the following.

Theorem 7 *There exists a matrix $F \in \mathbf{R}^{m \times p}$ such that $\alpha E - \beta(A + BFC)$ is a regular pencil of index ≤ 1 if and only if conditions* **C2** *and* **O2** *hold.*

Proof: See [18] and [4].

Corollary 8 *There exists a feedback control $u = Fy + v$ or $u_k = Fy_k + v_k$ such that $\alpha E - \beta(A + BFC)$ is a regular pencil of index ≤ 1 and the closed loop system defined by the triples $(E, A + BFC, B)$ and $(E, A + BFC, C)$ is S-controllable and S-observable if and only if the triples (E, A, B) and (E, A, C) are S-controllable and S-observable, respectively.*

Theorem 9 *There exists a matrix $G \in \mathbf{R}^{m \times p}$ such that $\alpha(E + BGC) - \beta A$ is a regular pencil of index ≤ 1 if the conditions* **C2** *and* **O2** *hold.*

Proof: See [4].

Corollary 10 *There exists a feedback control $u = -G\dot{y} + v$ or $u_k = -Gy_{k+1} + v_k$ such that $\alpha(E + BGC) - \beta A$ is a regular pencil of index ≤ 1 and the closed loop system defined by the triples $(E + BGC, A, B)$ and $(E + BGC, A, C)$ is S-controllable and S-observable if the triples (E, A, B) and (E, A, C) are S-controllable and S-observable, respectively.*

We remark that the converse of Theorem 9 does not hold. An example is given in [4].

In the following sections we consider the application of these results to the problems of pole placement and linear-quadratic optimal control for descriptor systems.

4 Pole Placement for Descriptor Systems

The problem of pole placement by derivative and proportional state feedback can be stated as follows.

Problem 1 *Given triple (E, A, B) and set $\mathcal{L} = \{\lambda_1, \lambda_2, \ldots \lambda_q\}$ where $\lambda_j \in \mathbf{C}$ and $\lambda_j \in \mathcal{L} \Rightarrow \bar{\lambda}_j \in \mathcal{L}, j = 1, 2, \ldots q \leq n$, find $F \in \mathbf{R}^{m \times p}, G \in \mathbf{R}^{m \times p}$ such that for some $X \in \mathbf{C}^{n \times q}$*

$$(A + BF)X = (E + BG)X\Lambda, \qquad \Lambda = diag\{\lambda_j\} \tag{25}$$

and

$$\det(\lambda(E + BG) - (A + BF)) \neq 0, \qquad \text{for some } \lambda \notin \mathcal{L}. \tag{26}$$

We remark that (25) guarantees that the prescribed poles are assigned by the feedback $u = Fx - G\dot{x}$ or $u_k = Fx_k - Gx_{k+1}$, and (26) ensures that the system pencil is regular.

For C-controllable systems we can assign a full set of n poles to the closed loop system by using a combination of proportional and derivative feedback. We have the following result.

Theorem 11 *For any arbitrary set $\mathcal{L}$ of n self-conjugate (finite) poles, there exists a pair of matrices F and G solving the pole placement problem, Problem 1, if and only if the triple (E, A, B) is C-controllable.*

Proof: The proof follows directly from Theorem 5 by selecting G such that $E + BG$ is nonsingular and then selecting F to assign the prescribed poles to the equivalent standard system (20). For details see [4].

We remark that with proportional state feedback alone we can assign at most $q = \operatorname{rank} E$ finite poles to the closed loop system. (The remaining $n - q$ infinite poles cannot be reassigned.) The use of derivative feedback alone allows us to reassign up to rank A poles, including all the infinite poles (but excluding the null poles $\lambda_j = 0$).

For S-controllable systems we can assign exactly $q \equiv \operatorname{rank} E$ poles with regularity of the system pencil. The closed loop system is then index ≤ 1 and regular. We have

Theorem 12 *For any set $\mathcal{L}$ of q self-conjugate (finite) poles, where $q \equiv \operatorname{rank} E$, there exists a solution to the pole placement problem, Problem 1, if and only if the triple (E, A, B) is S-controllable.*

Proof: See [18] and [4].

We remark that for S-controllable systems a closed loop system pencil which is regular and index ≤ 1 and has the prescribed poles can be achieved by proportional state feedback alone. We expect, however, that a more well-conditioned dynamic-algebraic system can be obtained by first using a derivative feedback to obtain a regular pencil of index ≤ 1 and then applying proportional feedback to assign the finite poles to specific positions.

We remark also that for the problem of pole assignment by output feedback in descriptor systems analogous results can be obtained.

5 Linear Quadratic Regulator Problems for Descriptor Systems

The linear quadratic optimal control problem for descriptor systems can be stated as follows

Problem 2 *Given triple (E, A, B) and $Q = Q^T \in \mathbf{R}^{n\times n}$, $R = R^T \in \mathbf{R}^{m\times m}$ and $S \in \mathbf{R}^{n\times m}$ such that*

$$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \geq 0 \qquad \textit{and} \qquad R > 0, \tag{27}$$

find $F \in \mathbf{R}^{m\times p}, G \in \mathbf{R}^{m\times p}$ such that $u = Fx - G\dot{x}$ or $u_k = Fx_k - Gx_{k+1}$ minimizes

$$C = \int_{t_0}^{t_f} \begin{bmatrix} x \\ u \end{bmatrix}^H \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} dt \tag{28}$$

subject to (1), or

$$C = \frac{1}{2}\sum_{k=0}^{K} \begin{bmatrix} x_k \\ u_k \end{bmatrix}^H \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \begin{bmatrix} x_k \\ u_k \end{bmatrix} \tag{29}$$

subject to (3).

If E is nonsingular, then the problem can be solved immediately. If E is singular and (14) is satisfied, that is, the triple (E, A, B) is C-controllable, then by Theorem 5 we can choose a matrix G such that $E + BG$ is nonsingular and such that the corresponding closed loop system is equivalent to the standard system (20) or (21) with feedback control u given by (22) or (23) where $C = I$. By Theorem 5, G can also be selected to ensure that $V = I - GS^{-1}B$ is nonsingular, and hence that

$$\tilde{Q} = \begin{bmatrix} I & 0 \\ W & V \end{bmatrix}^H \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \begin{bmatrix} I & 0 \\ W & V \end{bmatrix} \geq 0, \qquad V^H R V > 0, \tag{30}$$

where $W = -GS^{-1}A$. The LQR problem then reduces to finding a feedback F such that $v = Fx$ or $v_k = Fx_k$ minimizes

$$C = \int_{t_0}^{t_f} \begin{bmatrix} x \\ v \end{bmatrix}^H \tilde{Q} \begin{bmatrix} x \\ v \end{bmatrix} dt \tag{31}$$

or

$$C = \frac{1}{2}\sum_{k=0}^{K} \begin{bmatrix} x_k \\ v_k \end{bmatrix}^H \tilde{Q} \begin{bmatrix} x_k \\ v_k \end{bmatrix} \tag{32}$$

subject to (20) or (21). This problem is immediately solvable. We obtain the following.

Theorem 13 *A solution to the linear quadratic regulator problem, Problem 2, exists if the triple (E, A, B) is C-controllable. Furthermore, the closed loop system corresponding to the triple $(E + BG, A + BF, B)$ is then stable.*

Proof: See [4] for details.

By similar arguments, if E is singular and the triple (E, A, B) is S-controllable. then Theorem 7 implies that F can be chosen such that the closed loop system pencil $\alpha E - \beta(A + BF)$ is regular and index ≤ 1, and such that the triple $(E, A + BF, B)$ remains S-controllable. The LQR problem is then in the form required by [24] and a solution to the transformed problem can be found. We thus have the following.

Theorem 14 *A solution to the linear quadratic regulator problem, Problem 2, exists if (E, A, B) is S-controllable.*

Proof: See [24] and [4].

We remark that the latter problem is solved by proportional feedback alone. We expect, however, that the derivative feedback may also be used here to obtain a more well-conditioned dynamic-algebraic system to which the proportional feedback may be applied.

We remark also that similar constructions apply to the linear quadratic output control problems.

6 Conclusions

We investigate here the use of derivative and proportional feedback in descriptor, or generalized state-space systems. We define various conditions for controllability and observability and demonstrate to what extent the structure of the system pencil can be altered by proportional-derivative feedback under these conditions.

It is established that systems which are C-controllable and C-observable can be transformed into *standard* systems by a combination of derivative and proportional state or output feedback. It is shown that in this case, with state feedback, all of the poles of the system can be assigned to prescribed positions; furthermore, the linear quadratic optimal control problem can be solved explicitly to obtain a stable closed loop system.

It is also established that systems which are S-controllable and S-observable can be transformed by proportional-derivative state or output feedback into closed loop systems which are regular and of index ≤ 1. It is shown that by state feedback the maximum number of finite poles can be assigned and the linear quadratic regulator problem can also be solved.

Details of the proofs of these results and examples are given in [4].

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