The No Response Test for the Reconstruction of Polyhedral Objects in Electromagnetics

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Abstract

We develope a *No Response Test* for the reconstruction of some polyhedral obstacle from one or few time-harmonic electromagnetic incident waves in electromagnetics. The basic idea of the test is to probe some region in space with waves which are small on some test domain and, thus, do not generate a response when the scatterer is inside of this test domain.

This is the first formulation of the No Response Test for electromagnetics. We will prove *convergence* of the method for testing a non-vibrating domain B whether the far field pattern of some scattered time-harmonic field is analytically extendable into the interior of B. We will describe algorithmical realizations of the No Response Test. Finally, we will show the feasibility of the method by reconstruction of polygonal objects in three dimensions.

Key words: Electromagnetic Waves, Maxwell Equations, Inverse Scattering, Object Reconstruction, Sampling Method, No Response Test

1 Introduction

Using electromagnetic waves for probing and investigation of unknown regions in space is widely employed in the natural sciences, ranging from optics and microscopy via X-Ray science to radar and electromagnetic tomography. An introduction into the mathematical theory of inverse problems for acoustic and electromagnetic waves can be found in (Colton and Kress, 1998). A survey about several more recent methods is given by (Potthast, 2006) and a comparative study of some of these methods can be found in (Honda et al., 2007).

Our goal here is to formulate and analyse the *No Response Test* first suggested in acoustics by (Luke et al., 2003) for object identification in electromagnetics.

In particular, we will provide a convergence analysis for the reconstruction of a polygonal perfectly conducting object in three dimensions from the far field pattern of two incident time-harmonic electromagnetic waves.

Let D be a polyhedral domain in $\mathbb{R}^3$. We consider the following electromagnetic scattering problem. The propagation of time-harmonic electromagnetic fields in a homogeneous media is governed by the *Maxwell equations*

$$\text{curl } E - i\kappa H = 0, \quad (1)$$
$$\text{curl } H + i\kappa E = 0, \quad (2)$$

in $\mathbb{R}^3 \setminus \overline{D}$ where κ is the real positive *wave number*. At the boundary of the scatterers the total field E satisfies the *Dirichlet boundary condition*

$$\nu \times E = 0 \text{ on } \partial D. \quad (3)$$

We look for solutions of the form $E := E^i + E^s$, and $H = \frac{1}{i\kappa}\text{curl } E$, of (2) and (3) where the *scattered field* (E^s, H^s) is assumed to satisfy the Silver-Müller radiation condition

$$\lim_{r\to\infty} (H^s \times x - rE) = 0, \quad (4)$$

$r = |x|$ and the limit is uniform with respect to all the directions $\theta := \frac{x}{|x|}$, while the incident field (E^i, H^i) is given by

$$E^i(x, d, p) = \frac{i}{\kappa}\text{curl curl } pe^{i\kappa x\cdot d} = i\kappa(d \times p) \times de^{i\kappa x\cdot d},$$
$$H^i(x, d, p) = \text{curl } pe^{i\kappa x\cdot d} = i\kappa d \times pe^{i\kappa x\cdot d}, \quad (5)$$

where $d \in \mathbb{R}^3$ is the direction of incidence and $p \in R^3$ is the direction of propagation.

It is proven by Cakoni, Colton and Monk (Cakoni et al., 2004) that a solution to this problem exists and it is unique. In addition, from the classical theory as presented for example in (Colton and Kress, 1998), the scattered field satisfies the following asymptotic property,

$$E^s(x, d, p) = \frac{e^{i\kappa r}}{r}(E^\infty(\theta, d, p) + O(r^{-1})), \quad r \to \infty,$$
$$H^s(x, d, p) = \frac{e^{i\kappa r}}{r}(H^\infty(\theta, d, p) + O(r^{-1})), \quad r \to \infty, \quad (6)$$

where $(E^\infty(\cdot,d,p), H^\infty(\cdot,d,p))$ defined on the unit sphere $\mathbb{S}$ is called the far field pattern associated to the incident field $(E^i(\cdot,d,p), H^i(\cdot,d,p))$.

We will study and solve the shape reconstruction problem for polygonal domains.

Definition 1.1 (Shape reconstruction problem) *Given $E^\infty(\cdot,d,p)$ on $\mathbb{S}$ with N directions, $N \geq 1$ of incidence $d_i, i = 1, ..., N$ and polarization $p_j, j = 1, ..., M$ for the scattering problem (2) - (4) reconstruct the obstacle D.*

2 The No Response Test in Electromagnetics

2.1 The Idea of the No Response Test

We consider scattering of incident plane waves with direction of incidence d and with polarization p_i for $i = 1, 2$. We assume that we have

$$p_i \perp d, \quad i = 1, 2 \text{ and } p_1 \text{ and } p_2 \text{ are not co-linear } . \tag{7}$$

For every $g \in L^2(\mathbb{S})$, we set $v_g(x) := \int_{\mathbb{S}} e^{i\kappa\theta\cdot x} g(\theta) ds(\theta)$ to be the scalar Herglotz wave corresponding the density g.

Then we define

$$I(B) = \lim_{\epsilon\to 0} \Big\{ \sum_{i=1}^{2} \Big| \int_{\mathbb{S}} E^\infty(-\theta, d, p_i) g(\theta)\, ds(\theta) \Big| : \ |v_g|_{C^1(B)} \leq \epsilon \Big\} \tag{8}$$

for any nonvibrating domain B, i.e. B is in the set

$$\mathcal{B} := \Big\{ B : \begin{array}{l} \text{the homogeneous interior Maxwell problem for } B \text{ does} \\ \text{have at most the trivial solution} \end{array} \Big\} \tag{9}$$

The idea of the No Response Test is to test if the unknown obstacle D is included in some $B \in \mathcal{B}$ by computing $I(B)$. In the next subsection, we show how this idea can be used to reconstruct the convex hull of D.

2.2 Convergence of the NRT.

Our key goal is to prove the following reconstruction of the convex hull of D.

THEOREM 2.1 (NO-RESPONSE CHARACTERIZATION) *The convex hull of D is characterized by*

$$\overline{CH(D)} = \bigcap_{B \in \mathcal{B}, I(B)=0} B. \tag{10}$$

Further, as a consequence of this results we immediately obtain the following uniqueness statement.

Corollary 1 *The convex hull of a polygonal domain in $\mathbb{R}^3$ is uniquely determined by the scattered field for one ($N = 1$) directions of incidence and $M = 2$ polarizations.*

DEFINITION 2.2 (ADMISSIBLE VERTICES) *We call a convex vertex of ∂D admissible if we can continue at least one of the faces of ∂D to the infinity without crossing ∂D, again.*

We call a vertex an exterior convex vertex *if it is in the boundary $\partial CH(D)$ of the convex hull $CH(D)$ of D.*

REMARK 2.3 *The exterior convex vertices characterize the convex hull of D.*

We will need the following identity

$$E^\infty(\theta, d, p) = \frac{i\kappa}{4\pi} \int_{\partial D} \Big\{ \nu(y) \times E^s(y, d, p) + [\nu(y) \times H^s] \times \theta \Big\} e^{-i\kappa\theta\cdot y} ds(y) \tag{11}$$

given by using the Straton-Shu formula in $\mathbb{R}^3 \setminus \overline{D}$ for $E^s(\cdot, d, p)$, $H^s(\cdot, d, p)$ and $\Phi(\cdot, y)$ and their asymptotic behavior at infinity (see (Colton and Kress, 1998), Theorem 6.8) where ν is the outward normal of ∂D. Let $g \in L^2(\mathbb{S})$, then

$$\begin{aligned}\int_{\mathbb{S}} E^\infty(-\theta, d, p) g(\theta) ds(\theta) = \frac{1}{4\pi} \int_{\partial D} \Big\{ &- \nu(y) \times E^s(y, d, p) \times \operatorname{curl} v_g \\ &+ \frac{1}{i\kappa} [\nu(y) \times H^s] \times \operatorname{curl}\operatorname{curl} v_g \Big\}\, ds(y)\end{aligned} \tag{12}$$

Let $B \subset \mathbb{R}^3$ be a convex non-vibrating domain for the Maxwell equation, i.e. let the interior homogeneous boundary value problem with boundary condition $\nu \times E = 0$ be uniquely solvable. We consider two cases:

(A) $\overline{D} \subset \overline{B}$. Suppose that $|v_g| \leq \epsilon$, then from (12), we have , for $d = d_i$ and

$p = p_i^j$,

$$|\int_{\mathbb{S}} E^\infty(-\theta, d, p) g(\theta) ds(\theta)| \le C\epsilon$$

with a uniform constant C. This implies that $I(B) = 0$.

(B) $\overline{D} \not\subset \overline{B}$. In this case, we can find at least one exterior convex point of ∂D which is not in $\overline{B}$. We denote by z_0 one of these points. We consider a sequence of points z_q included in $\mathbb{R}^3 \setminus \overline{D}$ tending to z_0.

We consider the *multipole fields*

$$\psi_q := \frac{\epsilon}{2\beta(z_q, \mu_q)} (h_q \cdot \nabla_z)^{\mu_q} \Phi(x, z_q) \tag{13}$$

where h_q is a unit vector, μ_q is a multi-integer and

$$\beta(z_q, \mu_q) := \sup_{y \in \overline{B}} \{|(h_q \cdot \nabla_z)^{\mu_q} \Phi(x, z_q)|\}.$$

For every q we take $g_n^q \in L^2(\mathbb{S})$ such that $v[g_n^q]$ tends to ψ_q in $C^1(B \cup D)$.

From (12), we get:

$$\lim_{n\to\infty} \int_{\mathbb{S}} E^\infty(-\theta, d, p) g_n^q(\theta) ds(\theta) = \frac{1}{4\pi} \int_{\partial D} \Big\{ -\nu(y) \times E^s(y, d, p) \times \operatorname{curl} \psi_q + \frac{1}{i\kappa}[\nu(y) \times H^s] \times \operatorname{curl}\operatorname{curl} \psi_q \Big\} ds(y) \tag{14}$$

Using the Stratton-Chu formula and due to the form of ψ_q, we have:

$$\lim_{n\to\infty} \int_{\mathbb{S}} E^\infty(-\theta, d, p) g_n^q(\theta) ds(\theta) = \frac{\epsilon}{2\beta(z_q, \mu_q)} (h_q \cdot \nabla_z)^{\mu_p} E^s(z_q, d, p) +$$

$$\int_{\partial\Omega_R} \left\{ -\nu(y) \times E^s(y, d, p) \times \operatorname{curl} \psi_q + \frac{1}{i\kappa}[\nu(y) \times H^s] \times \operatorname{curl}\operatorname{curl} \psi_q \right\} ds(y)$$

where Ω_R is a ball of radius R large enough to contain $\overline{D}$. Arguing as in ((Colton and Kress, 1998), Theorem 6.6), we deduce that the integral over Ω_R tends to zero as R tends to infinity. Hence

$$\lim_{n\to\infty} \int_{\mathbb{S}} E^\infty(-\theta, d, p) g_n^q(\theta) ds(\theta) = \frac{\epsilon}{2\beta(z_q, \mu_q)} (h_q \cdot \nabla_z)^{\mu_q} E^s(z_q, d, p). \tag{15}$$

Lemma 2.4 (Extensiblility) *Assume that for some positive real number ρ, the set of vectors*

$$\Big\{ \sup_{|h|=1} \rho^\mu \frac{(h \cdot \nabla_z)^\mu E^s(z, d, p)}{\mu!}, \mu \in \mathbb{Z}_+ \Big\} \tag{16}$$

is uniformly bounded in a compact set V, where here the boundedness is understood componentwise. Then $E^s(z,d,p)$ is analytically extensible into an open neighbourhood $V_\rho = \{x : d(x,V) < \rho\}$ of V.

Proof of Lemma 2.4. The basic result can be found in (Honda et al., 2007) or (Potthast, 2007). The authors use (16) as a bound for the Taylor coefficients of the function and construct an analytic extension into the open neighbourhood of V by multi-dimensional Taylor series. □

LEMMA 2.5 *Consider the scattered fields $E^s(\cdot, d, p_i)$ for $i = 1, 2$ in a neigbourhood of an exterior c. Then there exists at least one pair (d, p_i) such that $E^s(z, d, p_i)$ is not analytically extensible into an open neighbourhood of the point z_0.*

Proof of Lemma 2.5. By definition of the exterior vertex, there exists at least one face around z_0 which can be extended to infinity without crossing again ∂D. On this face we have $\nu \times E = 0$. Since E is extendable near z_0 then it satisfies, with H, the Maxwell equations around z_0. Hence it is real analytic near z_0. This means that $\nu \times E = 0$ on an infinite part of the plan having as a normal ν. But $E = E^i + E^s$ and E^s tends to zero at infinity then we have $\nu \times E^i = 0$ on an infinite part of the plan. Recall that $E^i(x,d,p) = i\kappa(d \times p) \times d e^{i\kappa x\cdot d}$ hence

$$\lim_{|x|\to\infty} \nu \times (d \times p) \times d e^{i\kappa x\cdot d} = 0.$$

This implies that

$$\nu \times (d \times p) \times d = 0.$$

Since p is chosen orthogonal to d, then $(d \times p) \times d = p$ and hence $\nu \times p = 0$.

Having two polarization directions p_1 and p_2 orthogonal to d, then we get $\nu \times p_i = 0, i = 1, 2$, which means that ν is co-linear to both p_1 and p_2. But this contradicts the assumption that p_1 and p_2 are linearly indepedent. □

Corollary 2 *There exist sequences $(h_q) \subset \mathbb{S}$ and $(\mu_q) \subset \mathbb{N}$ such that*

$$\lim_{q\to\infty} \rho^{\mu_q} \frac{(h_q \cdot \nabla_z)^{\mu_q} E^s(z_q, d, p)}{\mu_q!} = \infty. \tag{17}$$

Proof of Corollary 2. It is a combination of Lemma 2.4 and Lemma 2.5. □

As it is shown in (Honda et al., 2007), the quantities β satisfy

$$|\beta(z_q, \mu_q)| \le C \frac{\mu_q!}{\rho^{\mu_q}}.$$

¿From (15) and Corollary 2, we have

$$\lim_{q\to\infty}\lim_{n\to\infty}|\int_{\mathbb{S}} E^{\infty}(-\theta,d,p)g_n^q(\theta)ds(\theta)|=\infty.$$

For $\epsilon>0$ fixed, we can take q,n large enough such that

$$\|v_{g_n^q}\|_{C^1(B)}\leq\|v_{g_n^q}-\psi_q\|_{C^1(B)}+\|\psi_q\|_{C^1(B)}\leq\epsilon.$$

This implies that $I(B)=\infty$. □

3 The Realization of the No Response Test

The basic goal of this chapter is to develop the numerical realization of the No Response Test. We will first describe general preparation steps which are uniform for all subsequent realizations of the No Response Test. Then, we will describe an efficient approach to realize the No Response Test numerically.

We consider an electromangetic *Herglotz wave function*

$$V[a](x):=\frac{i}{\kappa}\mathrm{curl}\,\mathrm{curl}\int_{\mathbb{S}} e^{i\kappa x\cdot\theta}a(\theta)\,ds(\theta),\ \ x\in\mathbb{R}^3 \tag{18}$$

with density $a\in T(\mathbb{S})$, where $T(\mathbb{S})$ denotes the set of all vector fields $a\in L^2(\mathbb{S})$ with $\nu(\hat{x})\cdot a(\hat{x})=0$ for all $\hat{x}\in\mathbb{S}$. Clearly, it satisfies the Maxwell equations (1) - (2). Further, consider the *magnetic multipole*

$$\Psi(x,z):=\frac{i}{\kappa}\mathrm{curl}\,\mathrm{curl}\,p\Phi(x,z),\ \ x\in\mathbb{R}^3 \tag{19}$$

with source point $z\in\mathbb{R}^3$. Now, let B be a non-vibrating domain in $\mathbb{R}^3$ with boundary of class C^2. Then, with the operator $\mathcal{H}:L^2(\mathbb{S})\to L^2(\partial B)$ defined by

$$(\mathcal{H}a)(x):=\frac{i}{\kappa}\mathrm{curl}\,\mathrm{curl}\int_{\mathbb{S}} e^{i\kappa x\cdot\theta}a(\theta)\,ds(\theta),\ \ x\in\partial B, \tag{20}$$

and $z\in\mathbb{R}^3\setminus\overline{B}$ we will study approximate solutions to the equation

$$\mathcal{H}a=\Psi(\cdot,z)\text{ on }\partial B. \tag{21}$$

With curl $_x(\varphi(x)a) = \text{grad}\,_x\varphi \times a$ when a does not depend on x we obtain

$$(\mathcal{H}a)(x) = i\kappa \int_{\mathbb{S}} e^{i\kappa x\cdot\theta}(\theta \times a(\theta)) \times \theta \, ds(\theta), \quad x \in \partial B, \tag{22}$$

and for tangential field $a(\theta) \in T(\mathbb{S})$ this reduces to

$$(\mathcal{H}a)(x) = i\kappa \int_{\mathbb{S}} e^{i\kappa x\cdot\theta} a(\theta) \, ds(\theta), \quad x \in \partial B, \tag{23}$$

First, we note important properties of equation (21).

Lemma 3.1 *The equation (21) does not have a solution $a \in L^2(\mathbb{S})$.*

Proof. Assume that there is a solution $a \in L^2(\mathbb{S})$ of equation (21). Then both fields $V[a]$ and $\Psi(\cdot, z)$ solve the Maxwell equations in B with identical boundary values. By the well-posedness of the interior Dirichlet problem in B the two fields will coincide in B. Now, since the fields are both analytic in $\mathbb{R}^3 \setminus \{z\}$, they coincide in $\mathbb{R}^3 \setminus \{z\}$. However, the field $V[a]$ is smooth in $\mathbb{R}^3$, but $\Psi(\cdot, z)$ has a singulity in z which is a contradiction. This proves the lemma. □.

We have shown that (21) does not have a solution. However, the operator $\mathcal{H}$ can be seen to have dense range in $L^2(\partial B)$.

Lemma 3.2 *The operator $\mathcal{H}$ defined by (20) is injective and has dense range as an operator from $T(\mathbb{S})$ into $L^2(\partial B)$.*

Proof. First, we study the injectivity of $\mathcal{H}$. Let $a \in T(\mathbb{S})$ be some density such that $Ha = 0$ on ∂B. Then, we have $V[a] \equiv 0$ in B due to the well-posedness of the interior Dirichlet problem for the Maxwell equations in B. Due to the analyticity of $V[a]$ in $\mathbb{R}^3$ we have $V[a] \equiv 0$ in $\mathbb{R}^3$. Now, we can apply Theorem 3.15 of (Colton and Kress, 1998) to conclude that $a = 0$. This proves injectivity.

To show the denseness of the range of $\mathcal{H}$ we consider the adjoint operator $\mathcal{H}^*$ which due to (22) is given by

$$(\mathcal{H}^*\psi)(\theta) = i\kappa \int_{\partial B} e^{i\kappa y\cdot\theta}\theta \times (\psi(y) \times \theta) \, ds(y), \quad \theta \in \mathbb{S}, \tag{24}$$

with $\psi \in L^2(\partial B)$. Assume that $\mathcal{H}^*\psi = 0$. Then according to (6.26) of (Colton and Kress, 1998) the function

$$W[\psi](x) := \text{curl curl} \int_{\partial B} \Phi(x,y)\psi(y)\, ds(y), \quad x \in \mathbb{R}^3 \tag{25}$$

has farfield $1/4\pi \cdot \mathcal{H}^*\psi = 0$. According to Rellichs lemma Theorem 6.9 of (Colton and Kress, 1998) the field $W[a]$ vanishes in $\mathbb{R}^3 \setminus \overline{B}$. We now pass to the tangental values of this field on the boundary via the vector jump relations (compare (2.86) in combination with Theorem 2.17 of (Colton and Kress, 1983)) and obtain

$$N\psi = \nu \times \text{curl curl} \int_{\partial B} \Phi(x,y)\psi(y)\, ds(y) = 0, \quad x \in \partial B. \tag{26}$$

This first needs to be carried out in an L^2 sense. Then we argue that solutions $\psi \in L^2$ of $N\psi = 0$ are continuous and use the uniqueness of the interior boundary value problem with homoneneous tangential boundary values and the classical jump relations to conclude that $\psi \equiv 0$ on ∂B. This ends the proof. □

As a consequence of the previous result we obtain that the equation (21) has approximate solutions in the sense that for every $\epsilon > 0$ there is $a \in T(\partial D)$ such that

$$\Big|\Big|\mathcal{H}a - \Psi(\cdot, z)\Big|\Big|_{L^2(\partial B)} \leq \epsilon. \tag{27}$$

In fact, the approximate solution to this equation can be calculated via classical Tikhonov regularization

$$a_\alpha := (\alpha I + \mathcal{H}^*\mathcal{H})^{-1}\mathcal{H}^*\Psi(\cdot, z), \tag{28}$$

which is equivalent to minimizing the functional

$$\mu[a] := ||\mathcal{H}a - \Psi(\cdot, z)||^2_{L^2(\partial B)} + \alpha||a||^2_{L^2(\mathbb{S})}. \tag{29}$$

Clearly, the minimum of the functional (29) tends to zero for $\alpha \to 0$ if $\mathcal{H}$ has dense range. Thus, via (28) we obtain stable approximate solutions for equation (21).

It has been shown in (Ben Hassen et al, 2006) that in fact we do not need to solve the full vectorial equation (21), but that it is sufficient to solve the scalar equation

$$Hg = \Phi(\cdot, z) \text{ on } \partial B_\rho \tag{30}$$

with some parameter $\rho > 0$, $B_\rho := \{x \in \mathbb{R}^3 : \ d(x, B) \leq \rho\}$ and

$$(Hg)(x) := \int_{\mathbb{S}} e^{i\kappa x\cdot\theta} g(\theta)\ ds(\theta), \quad x \in \mathbb{R}^m. \tag{31}$$

Then, the $a := p \cdot g(x)$ is a solution to (21). ¿From a algorithmical point of view to solve a scalar equation is clearly much more efficient. With the same arguments as above we can employ Tikhonov regularization for its solution, i.e. we calculate

$$g_{z,\alpha} := (\alpha I + H^*H)^{-1} H^* \Phi(\cdot, z) \text{ on } \partial B \tag{32}$$

for $\alpha > 0$. Also, it has been shown in (Ben Hassen et al, 2006) that by inserting the approximation of $\Phi(\cdot, z)$ into the Stratton-Chu formula we obtain an approximation

$$\int_{\mathbb{S}} E^\infty(\hat{x}) g_{z,\alpha}(\hat{x})\ ds(\hat{x}) \to E^s(z), \quad \alpha \to 0 \tag{33}$$

in the sense that given $\epsilon > 0$ there is $g_z \in L^2(\mathbb{S})$ such that

$$\Big| E^s(z) - \int_{\mathbb{S}} E^\infty(\hat{x}) g_z(\hat{x})\ ds(\hat{x}) \Big| \leq \epsilon \tag{34}$$

which holds under the condition that the field E^s can be analytically extended into $\mathbb{R}^3 \setminus B$.

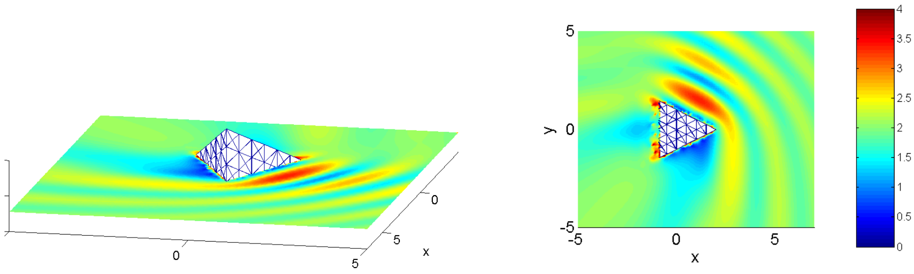
We now describe a direct realization of the No Response Test via the functional

$$I(B, d, p, \alpha) := \sup \Big\{ \Big| \int_{\mathbb{S}} E^\infty(-\theta, d, p) g(\theta) \ \ ds(\theta) \Big| : \ g \in \mathcal{G} \Big\} \tag{35}$$

for some nonvibrating domain B where $\mathcal{G}$ is some set of densities with

$$||v_g||_{C^2(B)} \leq \epsilon. \tag{36}$$

In particular, we will calculate such densities by solving the integral equation



(a) (b)

Fig. 1. Modulus of the total electric field for scattering by a polygonal domain with perfect conductor boundary condition, wave number $\kappa = 2$. We show two different views, (b) from above and (a) looking onto one of the edges. Half of the object is covered by the plane with the field visualization.

(30) and multiplying the solution with the constant c_ϵ which satisfies

$$c_\epsilon \leq \frac{\epsilon}{2||\Phi(\cdot, z)||_{C^2(B)}}. \tag{37}$$

Here, for simplicity we use $\rho = 0$.

Algorithm 3.3 (No Response Test via the NRT Functional) *The No Response Test estimates the functional (8) by calculating $I(B, d, p, \alpha)$ defined in (35), where for some domain B, a direction of incidence d and $\alpha > 0$ the density g is calculated by (30) for one or several points $z \in \mathbb{R}^3 \setminus \overline{B}$. In a second step we calculate the intersection*

$$D_{rec} := \bigcap_{I(B,d,p,\alpha) \leq c} B \tag{38}$$

with some adequate constant c.

We complete this work with some numerical reconstructions which prove the feasibility of the method. Figure 1 shows the simulation of the field via integral equation methods. We have tested the code by solving the exterior boundary value problem with a dipole with source point located in the interior of the object as reference field. The error was clearly below 2% even with a modest number of triangles as shown in Figure 1. Reconstructions are demonstrated in Figure 2. We show a visualization calculated via Algorithm 3.3 for different locations and sizes of the polygonal domain with wave numbers $\kappa = 1$.

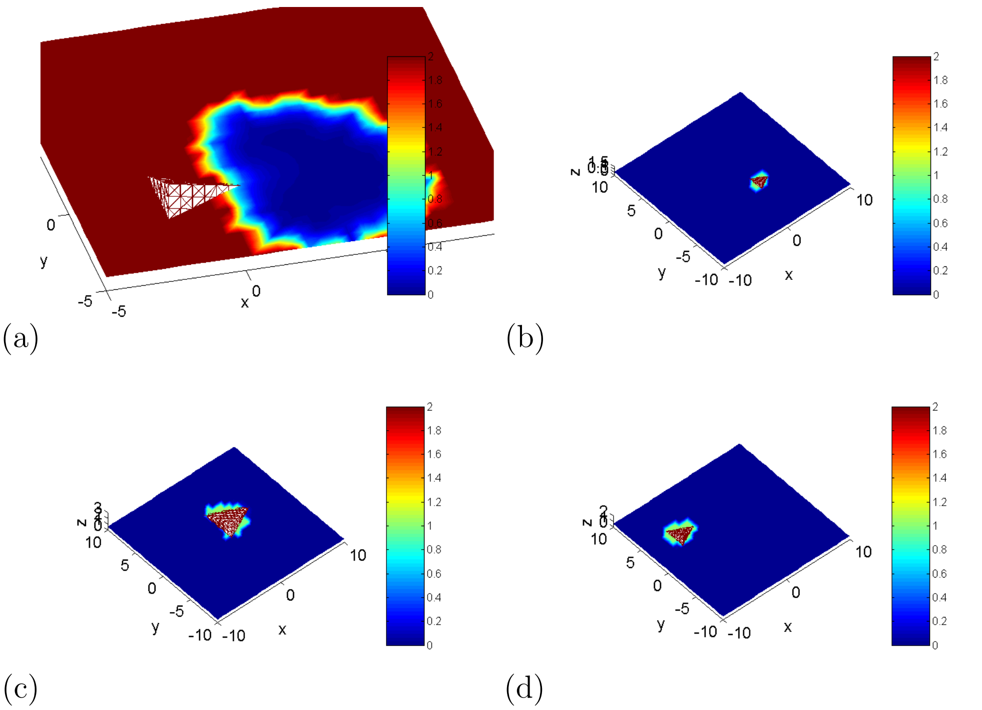


Fig. 2. In (a) we demonstrate the behaviour of the indicator function of the No Response Test for *one* electromagnetic wave only. Here, every image point z corresponds to a test domain $G(z)$ with $z \in \partial G_\rho(z)$ and $G(z) \subset \{y \in \mathbb{R}^3 : \; y_1 < z_1\}$. The blue area clearly indicates all such domains for which $D \subset G(z)$, i.e. it indicates a successfull No Response Test for the location of the domain. A second step is then to build the intersections (38). Figure (b) - (d) show reconstructions of some polygonal domain from the far field pattern of *one* wave via the No Response Test functional with balls as test domains. Here, we show a slice of the mask on a plane intersecting the scatterer. The results here have not been optimized to yield good shape reconstructions, but we worked on a grid with cells of size $h = 0.5$. Clearly, we can easily identify the location and size of the scatterer and prove the feasibility of the ideas described above.

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