

List Colouring Graphs with Functions

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The advent of graph list-colouring problems in the late 1970's has stimulated several distinct paths of inquiry. Here we follow one with numerous branches inspired by the realization of A. J. W. Hilton in 1988 that list-colouring problems on graphs have an ancestor in the theorem of Phillip Hall about systems of distinct representatives of finite families of finite sets. In the latest mutation along one of the branches descending from this realization, the "lists" are non-negative measurable functions on some fixed positive measure space, the vertices are supplied with positive real numbers, called "colour demands", and a proper colouring is an assignment to each vertex of a non-negative measurable "subfunction" of the list function at that vertex, with integral at least as large as the colour demand at that vertex, such that the product of the colour functions at any two adjacent vertices is zero. As usual, the first questions are: under what conditions on the graph, the measure space, the list assignment, and the colour demand will there be such a proper colouring?