Maximum size antichains and the COLEX order

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If *B* is a collection of sets we say the subset *A* of *B* is an antichain if for each pair of distinct sets in *A,* neither is a subset of the other. The order COLEX on the set of finite subsets of the positive integers is defined by C is less than D if C is a proper subset of D or if the largest element in C which is not in D is less than the largest element in D which is not in C. We find a formula for the maximum size of an antichain in the first m sets in COLEX. It is in terms of a certain new representation of m as a sum of binomial coefficients. We call it the Catalan cascade representation, because of connections with a generalization of Catalan numbers and with the traditional cascade representation of an integer. The special case when m is a power of 2 is Sperner's theorem on the maximum size of an antichain in the Boolean lattice. The formula points the way to an open conjecture which is an antichain analog of the Kruskal-Katona theorem, a fundamental result on the size of the "shadow" of a collection of sets.