Congruent and Similar Subsets in d-space

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Let $f_d(n)$ denote the maximum number of similar subsets that can occur among n points in d-space R^d, and let $g_d(n)$ be the maximum number of congruent subsets that can occur. Clearly $g_d(n) \leq f_d(n)$. A problem of Erdos and myself posed in 1975 asks to find an upper bound for $g_d(n)$, and we conjectured that $g_d(n) = O(n^{d/2})$. We discuss what is currently known about $g_d(n)$ and we show that

$$g_d(n) \leq f_d(n) = O(n^{d-\varepsilon}), \text{ where } \varepsilon = \varepsilon(d) > 0.$$

We also give a survey of related problems and results of Erdos and others. For example, how many congruent or similar triangles can occur in R^4 and R^5? How many congruent or similar simplices of dimension r can occur in R^d?