Dichotomy for the minimum cost graph homomorphism problem

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For graphs G and H, a mapping $f: V(G) \to V(H)$ is a homomorphism of G to H if $uv \in E(G)$ implies $f(u)f(v) \in E(H)$. If, moreover, each vertex $u \in V(G)$ is associated with costs $c_i(u), i \in V(H)$, then the cost of the homomorphism f is $\sum_{u \in V(G)} c_{f(u)}(u)$. For each fixed graph H, we have the *minimum cost homomorphism problem*, written as MinHOM(H). The problem is to decide, for an input graph G with costs $c_i(u)$, $u \in V(G), i \in V(H)$, whether there exists a homomorphism of G to H and, if one exists, to find one of minimum cost. Minimum cost homomorphism problems encompass (or are related to) many well studied optimization problems. We describe a dichotomy of the minimum cost homomorphism problems for graphs H, with loops allowed. When each connected component of H is either a reflexive proper interval graph or an irreflexive proper interval bigraph, the problem MinHOM(H) is polynomial time solvable. In all other cases the problem MinHOM(H) is NP-hard. This solves an open problem from an earlier paper.

(joint work with Pavol Hell, Arash Rafiey and Anders Yeo)