VERTEX-TRANSITIVE GRAPHS: SEMIREGULAR AUTOMORPHISMS AND HAMILTON PATHS

Dragan Marušič (Ljubljana and Primorska)

In view of this special occasion it seems only appropriate for my talk to reflect on my Reading years and my work done here as a PhD student of Crispin Nash-Williams and later on as a Research Fellow.

Time permitting, my intention is to discuss some recent developments on two of my favorite open problems in vertex-transitive graphs. Both happened to be the topics of my very first real talk given at the Reading University Combinatorics Seminar back in 1979, and, of course, my PhD thesis. The first one, a classical problem of Lovász which goes back to 1969 and asks if it is true that every connected vertex-transitive graph contains a Hamilton path, needs no special introduction. I will briefly describe a new approach that may be used to assure existence of a Hamilton path, for example, in every cubic Cayley graph arising from a group $G = \langle a, b | a^2 = 1, b^s = 1, (ab)^3 = 1, etc. \rangle$ with a $(2, s, 3)$-presentation. The second problem, appearing first in a written form in my PhD thesis in 1981, asks if it is true that every vertex-transitive graph has a semiregular automorphism, that is, a nonidentity automorphism with all orbits of equal size. Again, some recent results on this problem will be discussed together with a method for describing imprimitivity block systems of transitive permutation groups containing semiregular abelian subgroups. This method has proven particularly useful in obtaining classification results for certain families of low valency arc-transitive graphs.

Some of the research discussed in this talk comes out of joint work with Edward Dobson, Henry Glover, Istvan Kovacs, Aleksander Malnič, Štefko Miklavič and Lewis Nowitz.