

Edge weights and vertex colours

Andrew Thomason (Cambridge)

Given a graph, is it possible to assign an integer from the set $\{1,2,3\}$ to each edge so that, by then assigning each vertex the sum of its incident edge values, we get a proper vertex colouring?

Clearly not if the graph consists of a single edge, or has a component which is a single edge. It is conjectured that in every other case the answer is “yes”. This simple and universal conjecture remains open, to the best of my knowledge.

Some ideas will be described that lead to proofs of weak versions of the conjecture. The work is joint with Karonski and Luczak, and with Addario-Berry, Dalai, McDiarmid and Reed.