Iterative B-spline Neural Networks for Stochastic Distribution Control and Its Application in Industrial Process

H. Wang and H. Yue

***Abstract*—Iterative learning of B-spline basis functions model for the output probability density function (PDF) control of non-Gaussian systems is studied in this paper using the recursive least square algorithm. Within each control interval, the basis functions are fixed and the control input design is performed that controls the shape of the output PDFs. However, between each control interval, periodic learning techniques are used to tune the shape of the basis functions. This has been shown to be able to improve the accuracy of the B-spline approximation model. As such, the overall B-spline model of the output PDFs becomes a dual-model related to both time and space variables. The algorithm has been applied to a simulation study of the molecular weight distribution (MWD) control of a styrene polymerization process, leading to some interesting results.**

I. INTRODUCTION

IN recent years, the control of the whole shape of the output probability density function (PDF) has been studied in response to the increased demand from many practical systems ([1-21]). For this type of systems, the actual controlled output is the shape of the output probability density functions and the inputs are only related to time (such as flow rate and valve opening, etc). In this regard, the following partial differential equation (PDE) can be generally used to represent the dynamical evolution of the output PDFs

$$0 = \xi\left(\frac{\partial^n \gamma}{\partial y^n}, \frac{\partial^{n-1}\gamma}{\partial y^{n-1}}, \cdots, \frac{\partial \gamma}{\partial y}, \gamma, \frac{\partial^m \gamma}{\partial t^m}, \frac{\partial^{m-1}\gamma}{\partial t^{m-1}}, \cdots \frac{\partial \gamma}{\partial t}\right) \qquad (1)$$

where $\xi(\cdots)$ is a general nonlinear function and

The authors are with the Control Systems Centre, School of Electrical and Electronic Engineering, The University of Manchester, Manchester, UK. The authors are also affiliated with the Institute of Automation, Chinese Academy of Sciences, Beijing, P. R. China (e-mail: hong.wang@manchester.ac.uk).

$\gamma(\cdot)$ denotes the output PDF. This PDE model is a general expression of many population balance equations such as the following widely used particulate system model [20]

$$\frac{\partial W(\zeta,t)}{\partial t} + \frac{\partial[k(\zeta,t)W(\zeta,t)]}{\partial \zeta} = h(\zeta,t) \qquad (2)$$

where t is time; ζ is the internal coordinate; $W(\zeta,t)$ is the number density of particles; $k(\zeta,t)$ is the particle growth rate; and $h(\zeta,t)$ is the net creation of particles.

For the systems represented by either (1) or (2), the aim of the controller design is to ensure that the shape of the output probability density function can follow a target distribution. This type of control is termed as the stochastic distribution control (SDC), which is a new research area. In comparison with the traditional stochastic control theory where only output mean and variances are of concern, stochastic distribution control can offer a much better solution and are of course not restricted to Gaussian input cases. Indeed, this is a challenging problem and such systems are seen in general material processing industries.

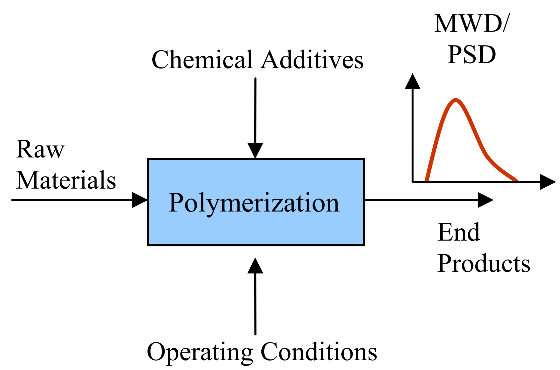

Figure 1 MWD/PSD in polymerization

Typical examples are particle size distribution (PSD) or molecular weight distribution (MWD) control in chemical engineering (see Fig. 1) and food processing shown in Fig 2, combustion flames distribution control displayed in Fig 3 and 2D paper web grammage distribution control in papermaking as shown in Fig. 4, etc. With the fast development of sensing technology, the output probability density functions of these systems are now becoming measurable. This has provided an excellent opportunity for control engineers to develop for the first time direct feedback control for the output probability density functions, leading to a much improved system performance.

In Fig. 1, the system is subjected to the inputs from the raw materials, chemicals and the required operating conditions such as temperature and pressure etc. The idea is to use chemical input for the chemical and physical reactions so that the produced materials would have a desired PSD/MWD which is characterized by its probability density functions ([9-18]).

As for the system represented in Fig. 2, the system is subjected to the original wheat particles, where the gap of the two rollers can be adjusted so as to make sure that the broken wheat particles have a desired probability density function ([2-5]).

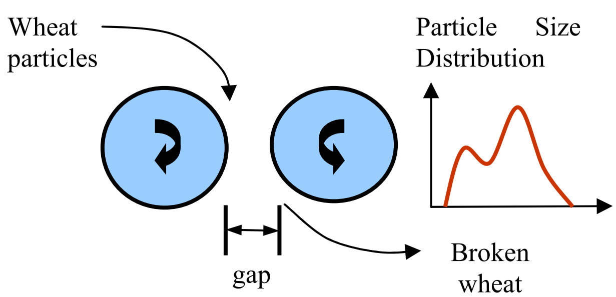

Figure 2. PSD in food processing

In Fig. 3, the general structure of a combustion csystem is given. The system is subjected to the fuel input together with some operating conditions, and produces a flames (or a temperature) distribution inside the combustion chamber. With the development of image processing, several digital cameras can be used to measure the distribution of the flame, which can be further transferred into the temperature distribution ([6-8]). An efficient combustion would mean that the distribution of temperature needs to be nicely controlled. This can also be formulated as to control the fuel flow rate so that the flames distribution can be made to follow a target distribution.

The system image represented in Fig. 4 reflects a 2D grammage distribution of a paper sheet ([1], [34], [35], [45]). This mimics a visual observation of someone holding a piece of paper against a strong light source. Paper is made of fibers together with other materials such as chemicals and fillers, etc. A good paper production would generally mean that the 2D distribution of the solids in the finished paper, per reflected by the grey level image in fig. 4, can be made as uniform as possible.

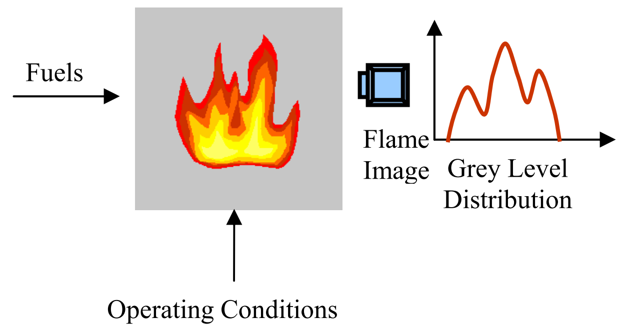

Figure 3. Flame distribution in combustion process

This is again an output probability density function control problem where the inputs are the chemical and mechanical variables in the paper machines whilst the output is the probability density function of the grey level distribution. In this regard, the target distribution should be characterized by a narrowly distributed Gaussian shape.

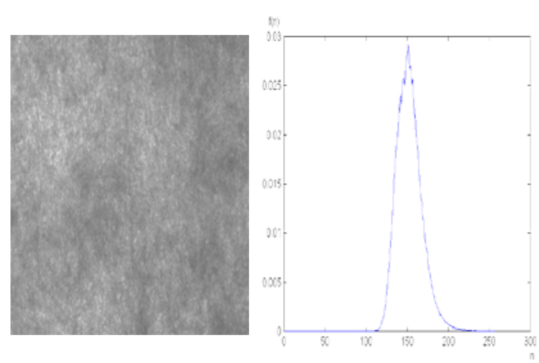

Figure 4 The grey level image of 2D paper web

The problem of controlling the output probability density function is long standing. However, the first practically implementable control strategy was developed at UMIST in 1996. Since then, fast developments have been seen and at present there are around 15 research groups in the world actively seeking solutions to the controller design and their applications. Special sessions have been seen in various international control conferences. In general, the so-far developed methods can be classified into the following three groups:

1) output probability density function control using neural networks;
2) output probability density function control using system input-output models;
3) output probability density function control using Ito differential equations.

Since 1998, many methods have been developed in this new research area. At present there are around 15 research centres worldwide actively seeking solutions and applications in this field. Invited sessions are seen in important international conferences and journals.

This paper will give a brief overview of the above developments and will then be followed by some detailed issues on how iterative learning mechanism can be combined with the B-spline based output probability density function control, where system modelling, controller design, basis function tuning, closed loop stability and learning convergence will be discussed. An application example will also be included.

II. A BRIEF OVERVIEW

The use of neural networks for the output PDF control is the first group of approaches, where once the structure of the network is selected the control of the PDF shape can be regarded as the control of the weights and biases of the considered network. In this context, a dynamical relationship can be established so as to link the network weights with the control input. As such, many existing control methods can be directly used to formulate the required control laws. In particular, when the dynamics is linear and the output PDF is approximated by a B-spline neural network, a compact solution can be generated which minimizes the following performance index

$$J(u) = \int_{\Omega} (\gamma(y,u) - g(y))^2 dy \quad + \; u^T Ru$$

where $\gamma(y,u)$ is the output PDF, u is the control input and R is a weighting matrix. The integration is calculated from a definition domain represented by Ω. Depending upon the dimensions of the system, the neural networks can be either MLP type ([48]) or even RBF types. As the above performance index is instant in terms of time, cumulative performance function of the following form

$$J = \sum_k \int_{\Omega} (\gamma(y,u_k) - g(y))^2 dy + u_k{}^T Ru_k$$

can also be used so as to guarantee the desired tracking performance of the closed loop system, where k is the sample time and the integration defines the difference between the actual output PDF and its target PDF. So far several B-spline based methods have been developed and examples are the square root B-splines PDF model and rational B-splines PDF model [22 - 33]).

However, the problem with this approach is that the model for the controller design does not have a direct physical meaning. Also, the network size can be very large if the output PDF shape is complicated, leading to a high dimensional dynamics between the network weights and the control input.

To solve this problem, a general input and output physical model of the system can be used. In discrete-time domain the following equation is employed.

$$y_k = f(y_{k-1}, y_{k-1}, \ldots, u_k, u_{k-1}, \ldots, \omega_k)$$

where y_k is the system output, u_k is the input, ω_k is a random process whose PDF is assumed known and f(…) characterizes the dynamics of the system. Using such a model, the output PDF can be formulated as a function of the PDF of the random input and all the past input and output measurements. As a result, a control input can be readily produced using a standard optimization routine for the performance function ([36-40]).

When a continuous time system is considered, the following Ito stochastic differential equation can be used to represent the system

$$dy = f(y,u)dt + g(y,u)d\omega$$

where y is the output, ω is a Brownian motion, f and g are nonlinear functions that represent the system dynamics and u is the control input. In this case a FPK partial differential equation can be formulated which belongs to the class of systems as shown in equation (1). This partial differential equation characterizes the dynamical behavior of the output PDF. As such, the control input can be designed so that the solution of the FPK equation can follow a target PDF. However, the difficulties are that the system has to be subjected to a Brownian motion before the Ito equation can be of any meaning. This limits the use of the method as only a

Gaussian dω input system can be considered.

Associated problems with the stochastic distribution control are the fault detection and diagnosis and minimum entropy control. These two aspects are also important in practice. In terms of fault detection and diagnosis ([41-42]), the idea would be to use the input and measured output PDFs to formulate effective fault diagnosis algorithms. As for the entropy control, the key feature is that the closed loop system should aim at minimizing its uncertainties (randomness) when a target PDF is not available. This can be achieved by using the formulation of minimum entropy tracking error control or simply the minimum entropy control of the system output ([43-47]).

III. ITERATIVE LEARNING IDEAS

Direct use of the PDE model is difficult in practice in that either such a model is difficult to establish through first principle approaches due to the complicated nature of the process, or the obtained control algorithms are too complicated to be applied in the real-time situations. To solve this problem, the B-spline approximation to $\gamma(\cdot)$ has been proposed since 1998 as one of the main groups of methods to control the output PDFs for non-Gaussian stochastic systems [22-25]. The idea is to use a set of fixed basis functions together with a group of time-varying weights to approximate the output PDFs at each time instant. The control input can therefore be designed to simply control the weights in the time-domain. This is equivalent to solving a PDE model by using the technique of separation variables with a fixed set of basis functions. That is, there are no space related differential equations in terms of the evolution of the shape of the basis functions. Several B-spline models have been developed ever since and have been shown capable of controlling the output PDFs to a good accuracy [26-32], albeit the number of B-spline basis functions can be quite high for complicated output PDF shapes and the accuracy to the PDF tracking may not be guaranteed. Since the PDF of a process can vary widely over operations, it may be difficult to capture the behaviour over an extended operating period with fixed basis functions. As a result, it would be ideal if the basis functions can be regularly updated according to the output PDF changes during the control process.

In the rest of this paper, the periodic learning and repetitive control are combined to perform the tuning of the basis functions for the output PDF control. In this context, the control horizon is divided into a number of intervals $\left[(j-1)(T+\Delta T), j(T+\Delta T)\right]$ $(j=1,2,\cdots)$ with T being the control interval length and ΔT being the time period to tune the B-spline basis functions. Within each interval $\left[(j-1)(T+\Delta T), jT+(j-1)\Delta T\right]$, the linear B-spline functions with FIXED basis functions are used to generate the required control inputs that control the output PDF shape. In the interval $\left[jT+(j-1)\Delta T, j(T+\Delta T)\right]$, the basis functions are updated to obtain a better approximation accuracy to the output PDFs. Such a set of updated basis functions will be used as the fixed basis functions for the next control interval. This means that the basis functions are tuned periodically and the following figure shows such a tuning phase.

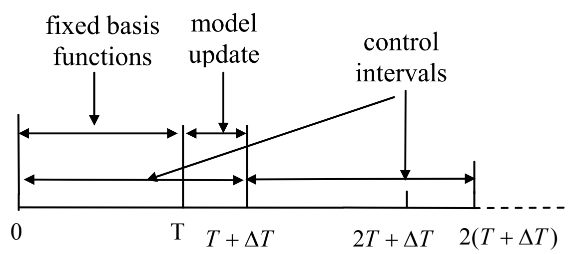

Figure 5. Illustrative tuning principle

To start with, in the period of [0,T] the control inputs are designed with a set of FIXED B-spline functions, whereby the control is realized via the control of the weights in the B-spline approximation. When the sample time reaches T, the tuning of the basis functions is activated. This will last for a period of ΔT, during which the control law stays the same as that of [0, T]. This enables the tuning to be focused on the basis functions and the parameters of the weights dynamics. Once the tuning is completed, the second control interval will start from the sample instant T+ΔT by using the updated basis functions and the model parameters. This process will repeat until the end of control horizon is reached. Different from existing iterative learning control methods, the proposed algorithm use the periodic learning for the basis functions and then tune the models batch by batch, thus achieving an improved performance over the whole time horizon.

IV. MODEL PRESENTATION

Assume that the output PDF of the considered stochastic systems in the *j*th tuning period is $\gamma_k(y,j)$, it is defined in a known interval denoted by [a, b] (i.e. $y \in [a,b]$). In this paper, the following linear B-spline function model [22] will be used to represent the dynamic relationship between the inputs and the output PDFs for each control instant k $\in [(j\text{-}1)(T+\Delta T), jT+(j-1)\Delta T]$

$$\begin{cases} V_k^j = G^j V_{k-1}^j + H^j u_{k-1}^j \\ \gamma_k(y,j) = C(y,j)V_k^j + L(y,j) \end{cases}$$

$$j = 1,2,\cdots; \quad k = 1,2,\cdots \tag{3}$$

where $V_k^j \in R^{n-1}$ is the weights vector that groups all the independent weights in the B-spline model; n is the number of basis functions chosen for approximation; u_{k-1}^j is a scalar input to the system; G^j and H^j are the parameter matrices which represent the system dynamics for k $\in [(j\text{-}1)(T+\Delta T), jT+(j-1)\Delta T]$. As presented in [22], let $B_i(y,j)$ (i = 1, 2, …, n) stand for the fixed basis functions for the *j*th control interval satisfying

$$b_i^j = \int_a^b B_i(y,j)dy \qquad i = 1,2,...n;\ j = 1,2,... \tag{4}$$

then in equation (3)

$$L(y,j) = \frac{B_n(y,j)}{b_n^j} \tag{5}$$

$$C(y,j) = \left[B_1(y,j) - \frac{b_1^j}{b_n^j}B_n(y,j), \quad B_2(y,j) - \frac{b_2^j}{b_n^j}B_n(y,j), \right.$$
$$\left. \cdots, \quad B_{n-1}(y,j) - \frac{b_{n-1}^j}{b_n^j}B_n(y,j) \right] \tag{6}$$

To simplify the following expression, denote

$$f_k(y,j) = \gamma_k(y,j) - L(y,j) \tag{7}$$

then the model in (3) can be further expressed in a one-step-ahead input and output form to read

$$f_{k+1}(y,j) = a_1^j f_k(y,j) + \cdots + a_{n-1}^j f_{k-n+2}(y,j) + C(y,j)D_0^j u_k^j$$
$$+ C(y,j)D_1^j u_{k-1}^j + \cdots + C(y,j)D_{n-2}^j u_{k-n+2}^j \tag{8}$$

a_i^j ($i = 1,\cdots,n-1$) and D_l^j ($l = 0,\cdots,n-2$) are the parameters and parameter vectors formulated from the state space model of equation (3) using information of G^j and H^j for k $\in [(j\text{-}1)(T+\Delta T), jT+(j-1)\Delta T]$. Similar to G^j and H^j, a_i^j and D_i^j are fixed within each control interval. However, their values are updated simultaneously with the tuning of the basis functions. The following performance function is used to measure the functional distance between the output PDF and the target PDF $g(y)$ (also defined on [a, b])

$$J^j = \int_a^b (\gamma_{k+1}(y,j) - g(y))^2 dy \tag{9}$$

where $\gamma_{k+1}(y,j)$ is the output PDF of the stochastic system at time instant $k+1$ for the *j*th control interval. To minimize J^j, u_k^j can be obtained by solving

$$\frac{\partial J^j}{\partial u_k^j} = 0 \tag{10}$$

to give the following feedback format

$$u_k^j = \frac{\int_a^b C(y,j)D_0^j \tilde{g}(y,j)dy}{\int_a^b (C(y,j)D_0^j)^2 dy} \tag{11}$$

where

$$\tilde{g}(y,j) = -\sum_{i=2}^{n-1}\left(a_i^j f_{k-i+1}(y,j) + C(y,j)D_{i-1}^j u_{k-i+1}^j\right) - L(y,j)$$
$$+ g(y) - a_1^j f_k(y,j) \tag{12}$$

This control law can be used together with the measured output PDFs so as to formulate a set of necessary information for the update of the basis functions as well as a_i^j and D_l^j within the *j*th tuning period $\left[jT+(j\text{-}1)\Delta T, j(T+T\Delta)\right]$.

V. BASIS FUNCTIONS AND { a_i^j, D_l^j } UPDATE

Assume that the control function (11) is applied to the system for k$\in [(j\text{-}1)(T+\Delta T), jT+(j-1)\Delta T]$, then according to Fig. 5 the update of the basis functions and { a_i^j , D_l^j } should take place in $\left[jT+(j\text{-}1)\Delta T, j(T+\Delta T)\right]$. For such an update, the information available should be

$$\Omega = \{f_{k+1}(y,j), a_i^j f_{k-i+1}(y,j), D_0^j u_k^j, D_1^j u_{k-1}^j, \cdots, D_{n-2}^j u_{k-n+2}^j\} \tag{13}$$

It is also important to note that in $\left[jT+(j\text{-}1)\Delta T, j(T+\Delta T)\right]$, the control inputs are still calculated using (11) with the same set of basis functions and parameters { a_i^j , D_l^j } for the *j*th control interval $[(j\text{-}1)(T+\Delta T), jT+(j-1)\Delta T]$, where Ω should satisfy equation (8).

Denote

$$T_k(y,j) = f_{k+1}(y,j) - \sum_{i=1}^{n-1} a_i^j f_{k-i+1}(y,j) \in R^{1\times 1} \quad (14)$$

$$\Pi_k^j = \left[\Pi_k^j(1) \quad \Pi_k^j(2) \quad \cdots \quad \Pi_k^j(n)\right]^T \in R^{n\times 1} \quad (15)$$

$$\pi(y,j) = [C(y,j) \quad L(y,j)] \in R^{1\times n} \quad (16)$$

$$\Pi_k^j(i) = D_0^j(i)u_k^j + D_1^j(i)u_{k-1}^j + \cdots + D_{n-2}^j(i)u_{k-n+2}^j, \quad i = 1,2,\cdots,n-1 \quad (17)$$

$$\Pi_k^j(n) = 1 \quad (18)$$

where $D_l^j(i)\,(l = 0,1,\cdots,n-2)$ is the ith component of vector D_l^j. Using the above notations and by fixing $\{a_i^j, D_l^j\}$, equation (8) becomes

$$T_k(y,j) = \pi(y,j)\Pi_k^j \in R^{1\times 1} \quad (19)$$

This is a linear model where the update of $\pi(y,j)$ should take place by using the data collected during [(j-1) (T+ΔT), jT+ (j-1)ΔT]. Similar to the scanning parameter estimation technique used in [15], a set of y_p are selected from the [a, b] interval for p = 1, 2, …., M, so that the following equation hold for each y_p

$$T_k(y_p,j) = \pi(y_p,j)\Pi_k^j, \quad p = 1,2,....,M \quad (20)$$

where M is a pre-specified positive integer. As $T_k(y_p,j)$ and Π_k^j are available at jth interval, $\pi(y_p,j+1)$ for the (j+1)th control interval can be directly updated by using a standard least square identification, leading to the following recursive least square algorithm:

$$\pi(y_p,j+1)^T|_{k+1} = \pi(y_p,j+1)^T|_k + \frac{P(k)\Pi_k^j e(k,y_p)}{1+\Pi_k^{jT}P(k)\Pi_k^j} \quad (21)$$

$$e(k,y_p) = T_k(y_p,j) - \pi(y_p,j+1)|_k \cdot \Pi_k^j \quad (22)$$

$$P(k) = P(k-1) - \frac{P(k-1)\Pi_k^j\Pi_k^{jT}}{\Pi_k^{jT}P(k-1)\Pi_k^j}P(k-1) \quad (23)$$

$$k = 1,\cdots,N; \quad p = 1,2,\cdots M$$

where N is the number of sampling points along the time axis [$(j-1)(T+\Delta T)$, $jT+(j\text{-}1)\Delta T$]; $\pi(y_p,j+1)^T|_k$ is the updated value of $\pi(y_p,j+1)^T$ at sample instant k. The initial value of $\pi(y_p,j+1)$ is evaluated from the most recent fixed basis functions and $P(0) = 10^{3-6}I_n$.

The procedures for the update of the B-spline functions (namely in the form of $\pi(y_p,j+1)$) is therefore given by:-

1. At sample time k, collect $T_k(y,j)$ and Π_k^j at the j th control interval;
2. Use equations (21)-(23) to calculate $\pi(y_p,j+1)$ with $p = 1,2,...M$;
3. Increase k by 1 and go back to step 1 until $k = N$. Here N is the number of data pairs sampled at each control interval.

Once the basis functions are updated, the next scan for the jth interval should be implemented to update the model parameters $\{a_i^j, D_l^j\}$. This can also be realized by the recursive least square algorithm. For this purpose, denote

$$\theta^j = \left[a_1^j,\cdots,a_{n-1}^j, D_0^j(1),\cdots,D_0^j(n-1),\cdots,D_{n-2}^j(1),\cdots, D_{n-2}^j(n-1)\right]^T \in R^{1\times(n^2-n)} \quad (24)$$

$$\phi(y,j,k) = \left[f_k(y,j),\cdots,f_{k-n+2}(y,j),u_k^jC_1(y,j),\cdots,u_k^jC_{n-1}(y,j), \cdots,u_{k-n+2}^jC_1(y,j),\cdots,u_{k-n+2}^jC_{n-1}^j(y,j)\right]^T \in R^{(n^2-n)\times 1} \quad (25)$$

where $\phi(y,j,k)$ is composed of the updated basis functions. As a result, the modification of θ^j is carried out using the following algorithm:

$$\theta^{j+1}(k+1) = \theta^{j+1}(k) + \frac{P_P(k)\phi(y_p,j,k)e_p(k,y_p)}{1+\phi(y_p,j,k)^T P_P(k)\phi(y_p,j,k)} \quad (26)$$

$$e_p(k,y_p) = f_{k+1}(y_p,j) - \theta^{j+1}(k)^T\phi(y_p,j,k) \quad (27)$$

$$P_p(k) = \left(I - \frac{P_P(k-1)\phi(y_p,j,k)\phi(y,j,k)^T}{1+\phi(y_p,j,k)^T P_P(k-1)\phi(y_p,j,k)}\right)P_P(k-1) \quad (28)$$

where $P_p(0) = 10^{3-6}I_{n\times(n-1)}$, the initial value of θ^{j+1} is the value of θ used in the j th control interval.

The procedures used to update the parameter vector θ^{j+1} can be summarized as follows:

1. At sample instant k (=1, 2,$\cdots$, N), formulate $f_{k+1}(y_p,j)$ and $\phi(y_p,j,k)$;
2. Calculate $\theta^{j+1}(k)$ for $p = 1,2,...,M$ with equations (26)-(28);
3. Increase k by 1 and go back to step 1 until $k = N$.

The control and tuning of basis functions and parameters can be illustrated in figure 6.

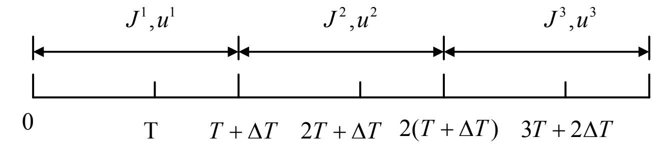

Figure 6. The control and parameters modification time series

The complete updating algorithm can be summarized as follows:

Step1: During [0, T], the closed-loop system uses a set of fixed basis functions and parameter vector θ to realize the control action as described in (11), where the data (namely u_k and $f_{k+1}(y)$) are stored for the updating operation of the basis functions;

Step2: From the time instant T to T+ ΔT , the saved data are used to calculate the B-spline basis functions with equations (21)-(23).

Step3. Using the updated B-spline basis functions in step 2, the saved data of [0, T] are used again to tune the model parameters via equations (26)-(28).

During step 2 and step 3, the system is controlled with the same model parameters and B-spline functions as those in [0, T];

The procedure will carry on until the pre-specified control horizon ends. This constitutes a periodic learning process, which regularly updates the basis functions and the model parameters for the weight dynamics.

VI. A SIMULATION STUDY OF MWD CONTROL

The proposed algorithm is applied to a simulation example of an MWD control system. The process of interest is a styrene bulk polymerization reaction in a pilot-plant continuous stirred tank reactor (CSTR) as shown in Fig. 7, in which styrene is the monomer for polymerization and azobisisobutyronitrile is used as the initiator. These two flows are injected into the CSTR with the ratio adjusted by a pump. The energy for the reaction is provided by the heated oil in the CSTR's jacket and the oil temperature is controlled to be constant. The total flow rate to the system, F , is composed of the flow of monomer, F_m , and the flow of initiator, F_i , i.e., $F = F_m + F_i$. The monomer input ratio is defined as $C = \frac{F_m}{F}$. In this work, adjustment of C is considered to be the means to control the MWD of the polymer. The model of this system can be seen in [33].

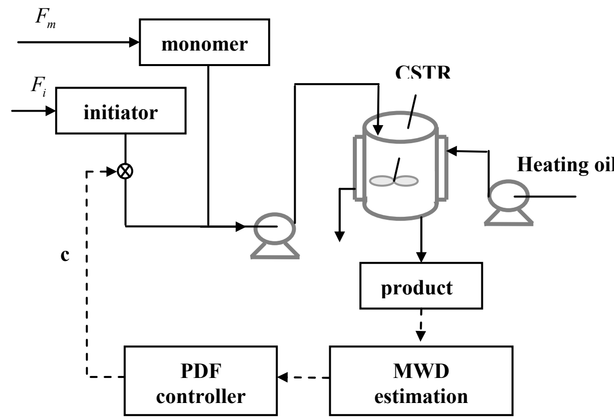

Figure 7. Styrene polymerization system in a pilot CSTR

For the above polymerization process, seven third-order polynomial B-spline functions are chosen for the MWD approximation. The B-spline model is established based on the data provided by a previously developed first-principle MWD model. The control input is designed so that the output MWD will follow a desired MWD. The periodic learning of the B-spline functions is carried out as illustrated in the previous section. Simulation results are shown in Fig. 8 to Fig. 11.

Fig. 8 shows the target MWD, the initial MWD and the output MWD of the system at the end of the control horizon, where it is clear that the output MWD can follow the shape of the target MWD. Fig. 9 displays the responses of the MWD in terms of a 3D mesh format, showing the periodic learning and batch-to-batch process. In this figure there are four control intervals, each consists of eighty MWD responses. In Fig. 10, the responses of the control input calculated from equation (11) are given, from which the periodic learning operation can be observed. For this system, the real control input to the process is limited in a range from 0.2 to 0.8. In Fig. 11, the closed-loop performance, namely Js_j

$$Js_j = \int_{(j-1)(T+\Delta T)}^{j(T+\Delta T)} \int_a^b (\gamma_{k+1}(y,j) - g_t(y))^2 dydt,$$

is displayed, indicating the consecutive improvement of the control results.

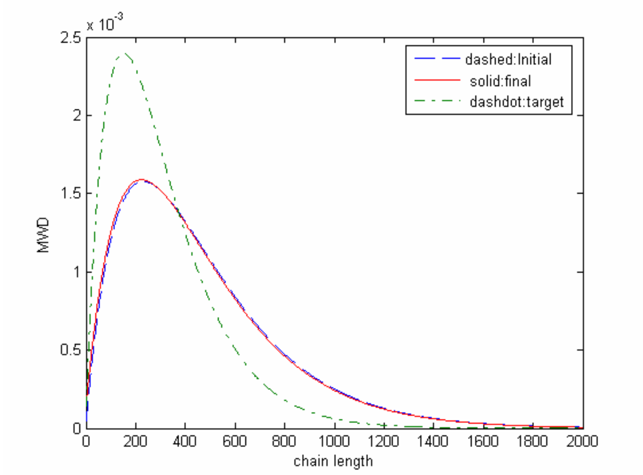

Figure 8. Initial, target MWDs and the output MWD at the end of control

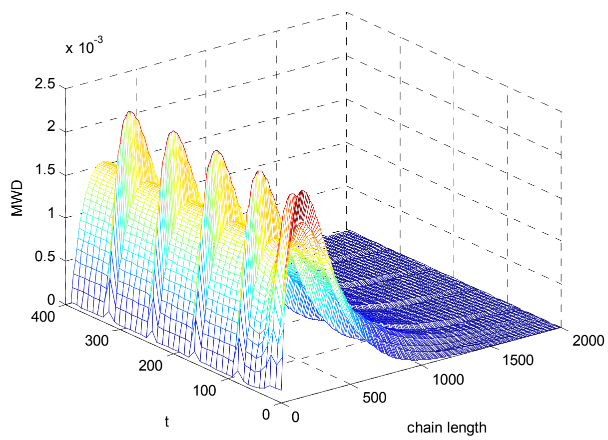

Figure 9. The output MWDs during the whole control process

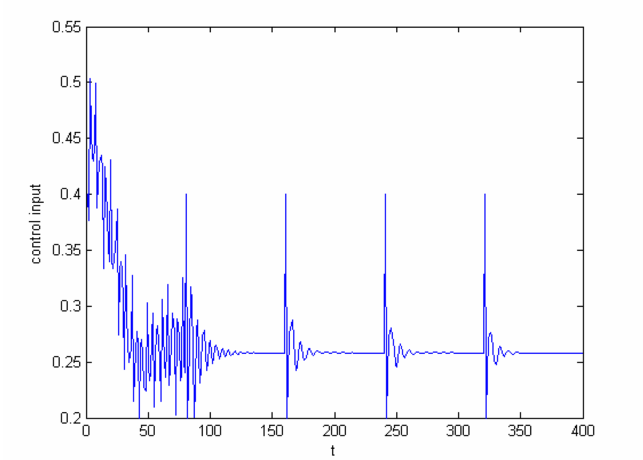

Figure 10. The control input during the control process

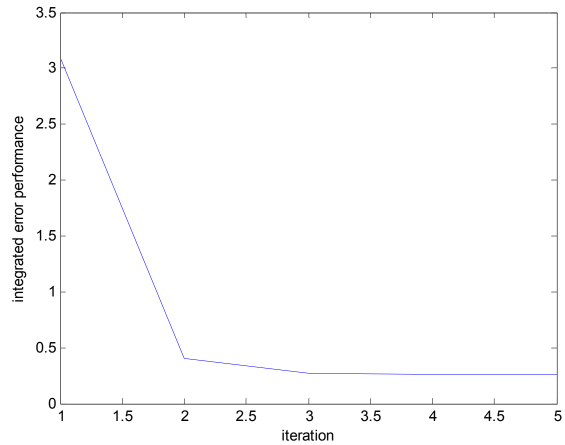

Figure 11. The integral error performance during iterations

As such, it can be seen from these figures that when applying the periodic learning algorithm to the MWD control of the polymerization system, the process has an improvement in terms of batch-to-batch operation.

VII. Alternative Iterative Learning for Model and Control Structure Reduction

To further explore the iterative learning ideas on the output PDF control, one can consider to use the following standard Radial Basis Functions (RBFs) neural network to model the instant output PDF as follows:

$$\gamma(y,u) \quad = \sum_{i=1}^{N} w_i(u)\phi(\frac{\| y - y_i \|}{\sigma_i}) \tag{29}$$

where w_i are the weights that are linked dynamically with the control input u(t) as they were before, y_i and σ_I are the parameters of the basis functions which represent the center and the width of each basis function $\phi(.)$. N is the number of basis functions used.

Between each batch, the iterative learning can be used to update the centre and width of the basis functions. For this purpose one can define the following parameter vector for the jth batch as that groups all the centers and the width as

$$\theta_j = [y_1, y_2, ..., y_N, \sigma_1, \sigma_2, ..., \sigma_N]^T \in R^{2N} \tag{30}$$

Define the closed loop control performance in the jth batch as follows:

$$J_j(u) = \sum_k \int_\Omega (\gamma(y,u_k) - g(y))^2 dy \quad + \quad u_k{}^T R u_k \tag{31}$$

then the following iterative learning rule can be generally obtained:

$$\theta_{j+1} = \theta_j + \lambda J_j \qquad (32)$$

where λ is a learning rate which can be either positive or negative as J_j is always positive here. This learning rate should guarantee convergence of the iterative leaning phase which is decided by the following condition

$$J_{j+1} < J_j \qquad (33)$$

which means that the tracking performance is improved along with the increase of the sub-interval index j. The advantage of this rule is that it can be used to minimize the model structure and thus have an impact on the control structure reduction. Indeed, if during the iterative learning phase one of the widths of the basis functions goes to a very small number, then this basis function can be removed from the output PDF approximation in (29). As a result, the weight dynamics can be reduced by at least 1 dimension. This will of course simplify the control structure, leading to a minimized controller structure for the closed loop system.

Another iterative learning rule can also be formulated using the standard control input updates. In this context, the control input is updated by the following rule

$$u_k^{j+1} = u_k^j + \lambda J_j \qquad (34)$$

In this case the learning rate should again be selected so that condition (33) is guaranteed. Different from the existing iterative learning control, this learning rate can be either positive or negative in the control horizon.

VIII. Conclusions

This paper presents an overview of the recently developed stochastic distribution control, where the B-spline based models are firstly discussed. This is then followed by the discussions on input and output based output PDF control and the Ito differential equation based algorithms.

In details, a periodic learning algorithm is proposed for the output PDF control based on the B-spline approximation model. With this control strategy, the output PDF model not only relates to time but also relates to space. With the update of the B-spline functions, the variation of PDF during operation can be considered and therefore the model accuracy of the PDF approximation can be improved. This algorithm is applied to the simulation study of a batch-to-batch MWD control system. Simulation results show the convergence and effectiveness of the algorithm.

The current method is only a periodic learning algorithm, in which the basis functions are updated by a least square identification rule. For a more effective update of the basis functions, certain iterative learning rules should be considered to modify the width and height of the basis functions. In the future work, the modification of the basis functions and the model parameters will be studied further to form a general expression for the output PDF control.

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